Notes on the Simulation of the Atmospheric Attenuation

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Abstract

In order to simulate the atmospheric attenuation suffered by the Cherenkov light emitted in the development of an atmospheric shower, an appropriate estimation of the quantities involved must be done. Since the classical CORSIKA simply gives us the height of emission of a photon in the vertical of the observer, the calculation of the true vertical height and the air mass traversed by the photons is of great importance.

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1 Calculation of the parameters

1.1 Calculation of the true vertical height

For the simple simulation of HZA using the plane-parallel-atmosphere version of CORSIKA, as well as for the calculation of the true vertical height of a photon at its emission point and the airmass (AM) it traverses, a very simple estimation can be done.

We will assume that our observer is located at point A (see Fig.1), at a height h_0 above sea level (a.s.l.). Our observer is looking at a source which emits gamma rays, hopefully, under a Zenith Angle θ. One of these gamma rays just arrived at the top of Earth's atmosphere, and developed an gamma ray induced atmospheric shower. (For simplicity, we will assume that the trajectory of the photon will cross the observation level at point A.)

We use the following notation:

A	Point where the observer is located
O	Point at sea level, in observer's vertical
P	Point of emission of the photon
h_0	Height of the observation level
h_{C}	Height given by CORSIKA, in the vertical of the observer
h_{v}	Height in the vertical of the photon
$R \equiv R_{\oplus}$	Earth's radius
θ	Zenith Angle in observer's level
ω	Zenith Angle at sea level, in observer's vertical
alpha	Angle from Earth's center between A and P

if $h_0 \neq 0$ nor negligible with respect to h_{C}, and $\theta simeq \omega$, the expression that gives us the value of the *true height* at P in the vertical, h_{v}, is:

$$h_{\mathrm{v}} = -R + \sqrt{(R+h_0)^2 + \left(\frac{h_{\mathrm{C}} - h_0}{\cos\theta}\right)^2 + 2(R+h_0)(h_{\mathrm{C}} - h_0)} \tag{1}$$

In the case of $h_0 \simeq 0$ or negligible with respect to h_{C}, this becomes:

$$h_{\mathrm{v}} \simeq -R + \sqrt{R^2 + \left(\frac{h_{\mathrm{C}}}{\cos\theta}\right)^2 + 2Rh_{\mathrm{C}}} \tag{2}$$

1.2 Pure geometrical estimation of the air mass

In the simplest approximation, the air mass factor (AM, the relation between the optical travel path at a given zenith angle and the optical path at the vertical) can be estimated from geometrical considerations. Using this simple approach, the air mass factor will be:

$$m \equiv \mathrm{AM} = \frac{\left[\sqrt{(R+h_{\mathrm{v}})^2 - [(R+h_0)\sin\theta]^2} - (R+h_0)\cos\theta\right]}{(h_{\mathrm{v}} - h_0)} \tag{3}$$

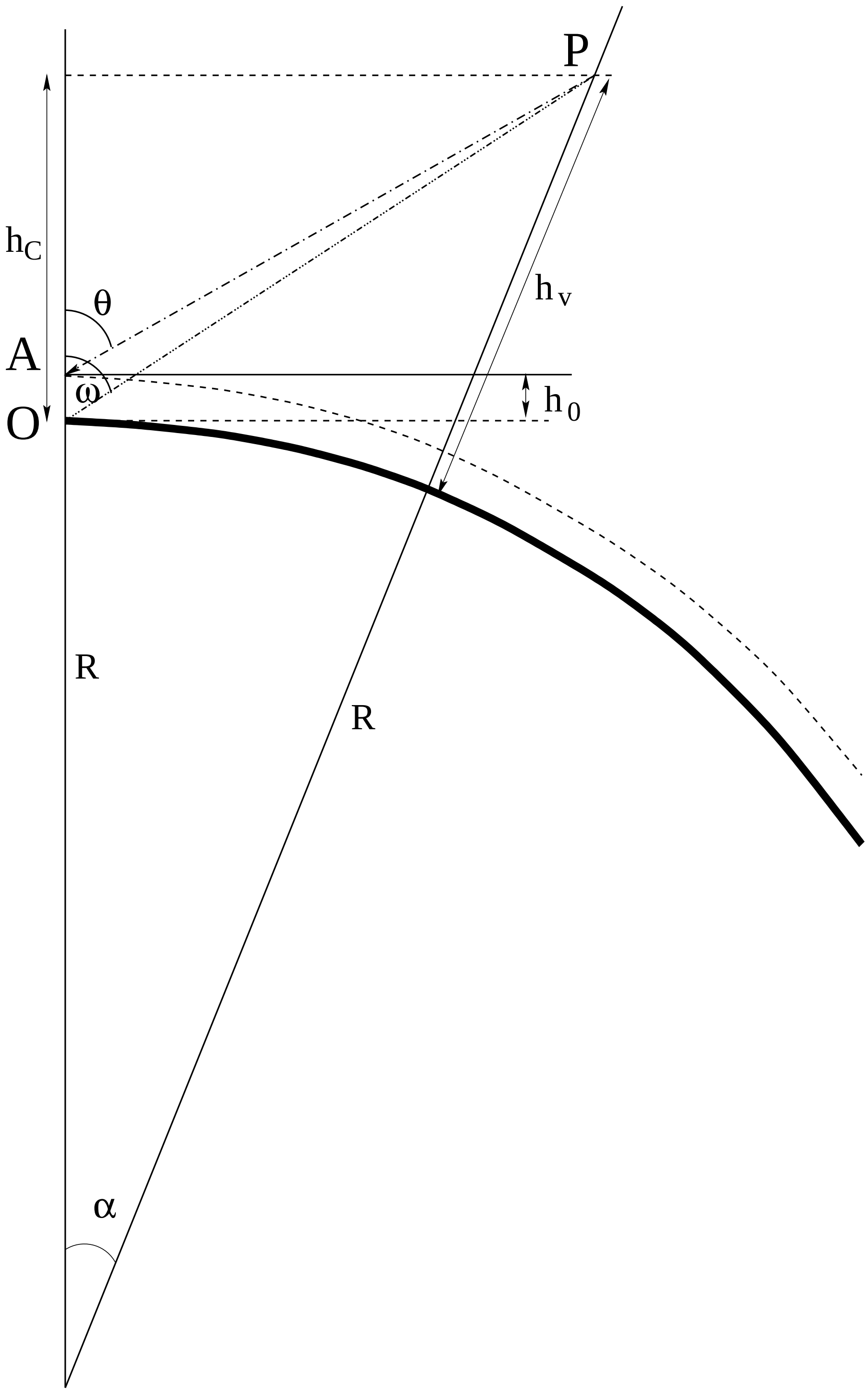


Figure 1: Simple illustration of the situation for an observer located at A, looking at a source under a Zenith Angle θ.

1.3 Calculation of the air mass using an exponential density model

The above expression in Eq.(3) uses just geometrical arguments to estimate the ratio represented by m. We would like to go one step beyond, by introducing an exponential density model. In our simulations we are using an atmospheric model represented by 4 exponentials and a linear function in the top of the atmosphere. Therefore, although the expression that we want to get is not fully correct, will be closer to the reality.

We will use then the following density model:

$$\rho(h) = \rho_0 \, e^{-h/H_\mathrm{S}} \tag{4}$$

where h is the height a.s.l., and H_S is the *scale height* of the atmosphere. In the Earth this results to be approximately $H_\mathrm{S} = 7.4\,\mathrm{km}$.

With this model, the optical path (ignoring refraction) is:

$$I(\theta; h_0, h_\mathrm{v}) = \int_{h_0}^{h_\mathrm{v}} \frac{\rho_0 \, e^{-h/H_\mathrm{S}} \left(1 + \frac{h}{R}\right) \mathrm{d}h}{\sqrt{\cos^2\theta + 2\left(1+\frac{h}{R}\right) + \left(1+\frac{h}{R}\right)^2}} \tag{5}$$

Dropping the terms in h/R,

$$I(\theta; h_0, h_\mathrm{v}) = \rho_o \sqrt{\frac{R}{2}} \int_{h_0}^{h_\mathrm{v}} \frac{e^{-h/H_\mathrm{S}} \mathrm{d}h}{\sqrt{h + \frac{1}{2} R \cos^2\theta}} \tag{6}$$

Note that we already wrote explicitly that we want to calculate the travel path for a Zenith Angle θ for vertical heights between h_0 and h_v.[1]

Integrating Eq.(6) we get:

$$I(\theta; h_0, h_\mathrm{v}) = \rho_o \sqrt{\frac{\pi R H_\mathrm{S}}{2}} \exp\left(\frac{R\cos^2\theta}{2\,H_\mathrm{S}}\right) \left[\mathrm{erfc}\left(\sqrt{\frac{2h_0 + R\cos^2\theta}{2\,H_\mathrm{S}}}\right) - \mathrm{erfc}\left(\sqrt{\frac{2h_\mathrm{v} + R\cos^2\theta}{2\,H_\mathrm{S}}}\right)\right] \tag{7}$$

where $\mathrm{erfc}(x)$ is the *complementary error function*. For the case of the vertical ($\theta = 0°$):

$$I(0°; h_0, h_\mathrm{v}) = \rho_o \sqrt{\frac{\pi R H_\mathrm{S}}{2}} \exp\left(\frac{R}{2\,H_\mathrm{S}}\right) \left[\mathrm{erfc}\left(\sqrt{\frac{2h_0 + R}{2\,H_\mathrm{S}}}\right) - \mathrm{erfc}\left(\sqrt{\frac{2h_\mathrm{v} + R}{2\,H_\mathrm{S}}}\right)\right] \tag{8}$$

[1] The *Airmass* in Optical Astronomy is usually defined as the relation:

$$\mathit{Airmass} \equiv \frac{I(\theta; 0, \infty)}{I(0°; 0, \infty)}$$

and therefore the value of our new air mass $\mathcal{AM}$ defined as:

$$\mathcal{AM} \equiv \frac{I(\theta; h_0, h_\mathrm{v})}{I(0^\circ; h_0, h_\mathrm{v})} \tag{9}$$

results:

$$\mathcal{AM} = \exp\left(\frac{-R\sin^2\theta}{2\,H_\mathrm{S}}\right) \frac{\left[\mathrm{erfc}\left(\sqrt{\dfrac{2h_0 + R\cos^2\theta}{2\,H_\mathrm{S}}}\right) - \mathrm{erfc}\left(\sqrt{\dfrac{2h_\mathrm{v} + R\cos^2\theta}{2\,H_\mathrm{S}}}\right)\right]}{\left[\mathrm{erfc}\left(\sqrt{\dfrac{2h_0 + R}{2\,H_\mathrm{S}}}\right) - \mathrm{erfc}\left(\sqrt{\dfrac{2h_\mathrm{v} + R}{2\,H_\mathrm{S}}}\right)\right]} \tag{10}$$

An approximation for low zenith angles, or more technically, for moderately large

$$X \equiv \sqrt{\frac{R\,\cos^2\theta}{2\,H_\mathrm{S}}} \tag{11}$$

is

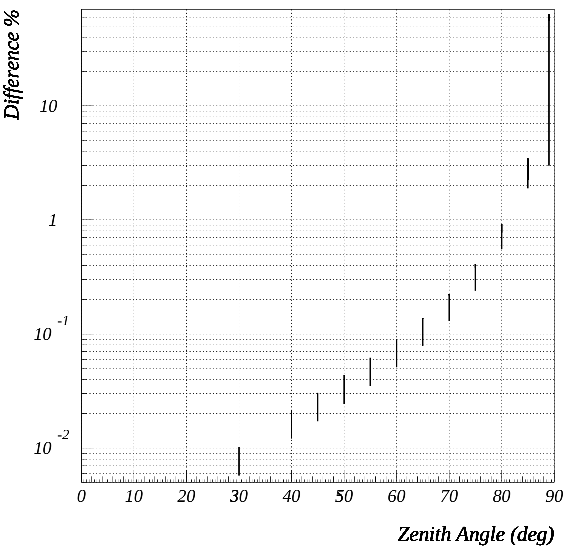
$$\mathcal{AM}_\mathrm{approx} = \sec\theta\left(1 - \frac{H_\mathrm{S}}{R}\sec^2\theta\right) \tag{12}$$

Correction for refraction. We have defined $R \equiv R_\oplus$, the radius of the Earth. A simple correction for refraction can be obtained by taking

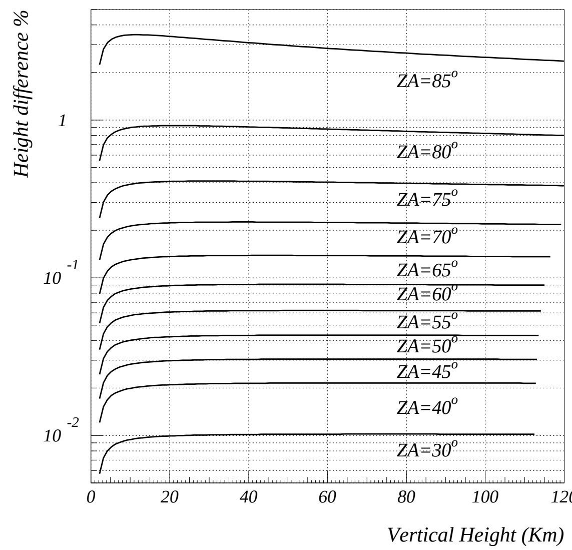
$$R = \frac{7}{6}R_\oplus \tag{13}$$

2 Comparison between different approximations

First, we would like to compare the true expression for the calculation of the h_v with the one obtained by assuming that h_0 is small enough. For this purpose we calculate the vertical height for $h_0 = 0$ km and $h_0 = 2.2$ km (height of the future location of MAGIC), for different Zenith Angles, and plot the difference in percentage (see Fig.2).



(a)



(b)

Figure 2: Difference of vertical heights for $h_0 = 0$ km and $h_0 = 2.2$ km: (a) as a function of the Zenith Angle, for different values of h_C; and (b) as a function of h_C, for different Zenith Angles.

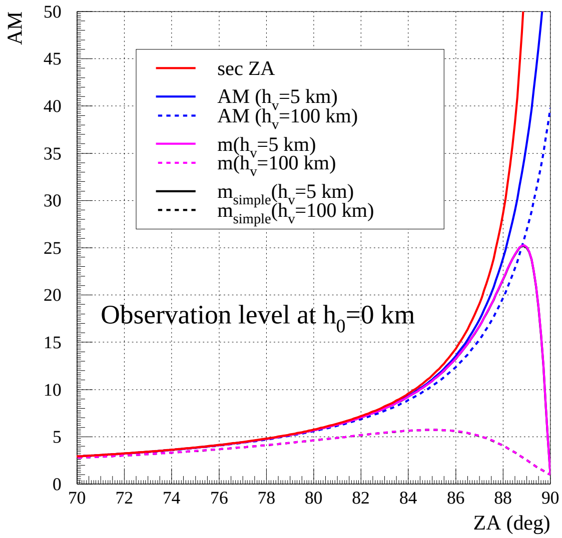
We can see that the difference increases with the zenith angle, as expected, but still the difference remains far below 5% up to 85°.

The second thing we want to compare is the geometrical expressions for the air mass, namely m and m_{simple} (the simple approximation for $ho = 0$ km), with the more elaborated calculation using the exponential density model, $mathcalAM$. We will include in the last expression the simple correction for refraction given in Eq.(13). This comparison, for three value of h_0 (0 km, 4 km and 2.2 km), is shown in Fig.3. We have used in this case two values for the vertical height, $h_v = 5$ km and $h_v = 100$ km.

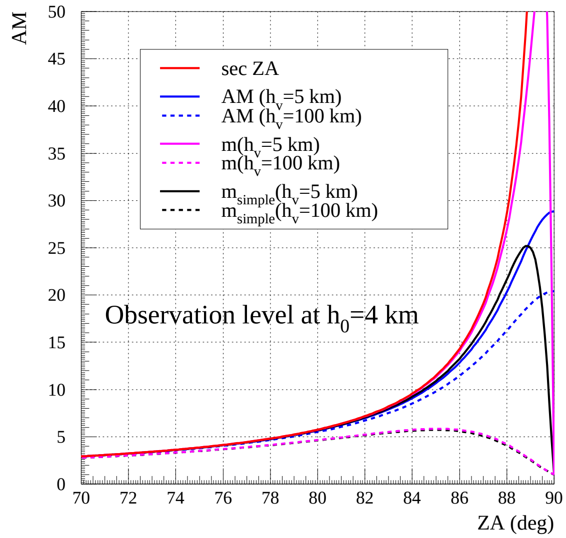
3 Calculation of the atmospheric transmission

Three effects have been included in the final calculation of the transmission of the atmosphere for the Cherenkov photons:

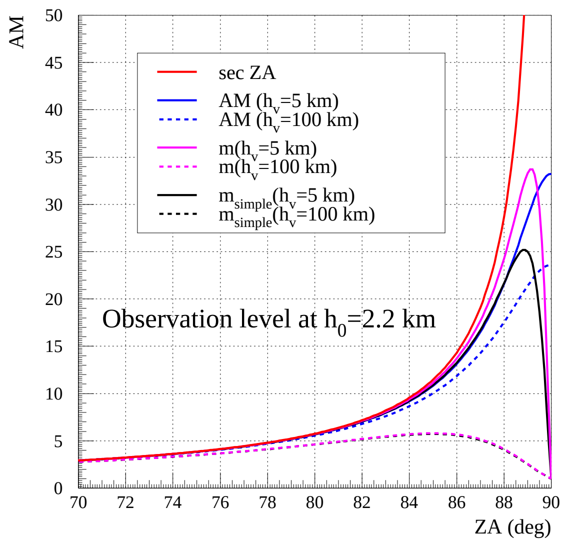
Rayleigh scattering. The molecules in the atmosphere produce scattering of the photons. This effect is strongly correlated with the wavelength λ of the light, being more important for small wavelengths. The transmission coefficient due to Rayleigh scattering is:



(a)



(b)



(c)

Figure 3: Comparison of m, m_{simple} and $\mathcal{AM}_{\text{refraction}}$, using two values for the vertical height ($h_{\text{v}} = 5\,\text{km}$ and $h_{\text{v}} = 100\,\text{km}$), as a function of the Zenith Angle (only values above 70° are shown), for three observation levels: (a) $h_0 = 0\,\text{km}$, (b) $h_0 = 4\,\text{km}$, and (c) $h_0 = 2.2\,\text{km}$.

$$T_{\mathrm{Rayl}} = \exp\left[-\frac{\tau_1 - \tau_2}{\tau_{\mathrm{R}}}\left(\frac{400\,\mathrm{nm}}{\lambda}\right)^4\right] \tag{14}$$

where $\tau_{i=1,2} = \tau_0 \exp(-h_i/H_{\mathrm{S}}) \sec\theta$ is the *slanted thickness* above a height h_i, in the case of an exponential density profile (actually in our case we calculated these values with the CORSIKA density model), $\tau_{\mathrm{R}}(\lambda = 400\,\mathrm{nm}) = 2970\,\mathrm{g/cm^2}$ is the mean free path of the Rayleigh scattering in terms of thickness, $\tau_0 = 0.00129\,\mathrm{g/cm^2}$, and H_{S} is the mentioned scale-height of the atmosphere. We pre-calculated the exponents for different wavelengths.

Mie scattering. This effect is due to the aerosol (dust) particles suspended in the air, being its effect bigger at lower heights. It depends also on the size of these particles. For an exponential density profile, the transmission coefficient is

$$T_{\mathrm{Mie}} = \exp\left[\frac{h_{\mathrm{M}}}{l_{\mathrm{M}}\cos\theta}\left(e^{\frac{h_1}{h_{\mathrm{M}}}} - e^{\frac{h_2}{h_{\mathrm{M}}}}\right)\right] \tag{15}$$

where $l_{\mathrm{M}}(400\,\mathrm{nm}) \simeq 14\,\mathrm{km}$ is the mean free path for the Mie scattering, and $h_{\mathrm{M}} \simeq 1.2\,\mathrm{km}$ is the scale-height for the aerosol distribution. We have pre-calculated the exponents for different wavelengths, although this dependence is very small.

Ozone absorption. This effect is very important in the range 280–340 nm and for low energy showers, in which most of the Cherenkov light emission is located where the density of ozone is high (between 20–30 km). For this we have first estimated the extinction in magnitudes per unit of air mass:

$$A_{\mathrm{Ozo}}(h_{\mathrm{v}}, \lambda) = 1.11 \cdot T(h_{\mathrm{v}}) k(\lambda) \tag{16}$$

where $k(\lambda)$ is the absorption coefficient in cm^{-1}, and $T(h_{\mathrm{v}})$ is the total ozone concentration for a typical Tropical region (we chose this because was very similar to the actual profile in La Palma), above the vertical in each position, in atmcm. The coefficient 1.11 was chosen to correspond to Elterman's optical thickness for ozone at 320 nm[3].

With this value, we can estimate the extinction in magnitudes per air mass between two height, and using the Pogson law we can determine the transmission coefficient, which results:

$$T_{\mathrm{Ozo}} = 10^{-[0.4\Delta A_{\mathrm{Ozo}}(h_{\mathrm{v}}, \lambda) m(h_{\mathrm{v}}, \theta)]} \tag{17}$$

For more details, please refer to [5].

Of course, the total transmission coefficient will be:

$$\mathcal{T} = T_{\mathrm{Rayl}} \cdot T_{\mathrm{Mie}} \cdot T_{\mathrm{Ozo}} \tag{18}$$

4 Conclusions

I hope that with this small document, the way we calculate the parameters for the simulation of the atmospheric attenuation is a bit more clear. Note that the more appropriate procedure to calculate the air mass for a given Zenith angle, between two vertical height, is, at the time of writing these note, *not* yet implemented in the simulation code.

The right equations should be used, when available and when the cost in computing time is inexistent with respect to any other more simple approach. However, from our studies it becomes clear that, under some conditions, simple approximations can give results that are accurate enough for most of the studies.

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