Notes on the estimation of the Accidental Trigger Rate due to the Light of Night Sky

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Abstract

The Artificial Trigger Rate (ATR) due to the Light of Night Sky (LONS) must be known in order to better understand the working conditions of our Cherenkov telescope. In this small note I try to estimate such ATR for different trigger conditions.

Light of the Night Sky

The amount of light from the night sky (LONS) has been measured, and is around

$$\langle \mathrm{LONS} \rangle = 2 \cdot 10^{12}\,\mathrm{ph/\,m^2\,s\,sr}$$

Estimation of the LONS per pixel for MAGIC

For this calculation, the following parameters have to be taken into account:

$S_{\mathrm{mirror}} = 230\,\mathrm{m}^2$ Reflectivity $= 80\%$
$\epsilon_{\mathrm{l.guides}} = 90\%$ $\epsilon_{\mathrm{plexiglas}} = 95\%$
$\epsilon_{1^{\mathrm{st}}\mathrm{dyn.coll.}} = 90\%$ $\theta_{\mathrm{1pixel}} = 0.1^\circ$
$\theta_{\mathrm{h.pixel}} = 0.05^\circ$ $\mathrm{QE_{LONS}} \sim 13\%$
$\Delta\Omega = 2\pi(1-\cos\theta_{\mathrm{h.pixel}}) = 2.39 \cdot 10^{-6}\,\mathrm{sr}$

Then, the mean number of photons arriving at the entrance of the pixel in 1 ns is:

$$\begin{aligned}
\mathcal{N}_{\mathrm{in}} &= \langle \mathrm{LONS} \rangle \cdot t \cdot S_{\mathrm{mirror}} \cdot \epsilon_{\mathrm{l.guides}} \cdot \epsilon_{\mathrm{plexiglas}} \cdot \Delta\Omega \\
&= (2 \cdot 10^{12}\,\mathrm{ph/\,m^2\,s\,sr}) \cdot (10^{-9}\,\mathrm{s/\,ns}) \cdot (230\,\mathrm{m}^2) \cdot (0.90) \cdot (0.95) \cdot (2.39 \cdot 10^{-6}\,\mathrm{sr}) \\
&= 0.94\,\mathrm{ph/\,ns}
\end{aligned}$$

Since our mean QE for the LONS is $\mathrm{QE_{LONS}} \sim 13\%$, this means:

$$\mathcal{N}'_{\mathrm{in}} = \mathcal{N}_{\mathrm{in}} \cdot \mathrm{QE_{LONS}} \cdot \epsilon_{1^{\mathrm{st}}\mathrm{dyn.coll.}} = 0.11\,\mathrm{ph.e^-/\,ns}$$

If we use then a gate of $\Delta T = 10\,\mathrm{ns}$, we arrive at a mean contribution of LONS per pixel per gate of:

$$\langle \mathrm{LONS} \rangle_{1\ \mathrm{pixel}} = \mathcal{N}'_{\mathrm{in}} \cdot \Delta T = 1.10\,\mathrm{ph.e^-/\,gate}$$

Estimation of the Number of Combinations

For the estimation of the Artificial Trigger Rate (ATR), we have to take into account several parameters:

1. The trigger pattern used for the trigger: it can be *simple multiplicity* (SM) of n pixels above a single pixel threshold, or a *next-neighbor* (NN) trigger of n pixels above the threshold, or any other pattern.

2. The single pixel threshold, in photoelectrons, milivolts, ...

3. The size of the camera, or more specifically, the size of the trigger region in the camera, and its topology.

In addition, of course, we have to include the rate of the LONS.
For the calculation of the ATR, then, we will use the following formula:

$$\mathrm{ATR} = \mathcal{N}_{\mathrm{comb}} R^n \Delta T^{(n-1)}$$

where R is the *singles* rate, that is, the trigger rate of a given individual pixel due to the LONS, n is the number of pixels involved in the trigger, ΔT is the time window fixed in our trigger logic, and $\mathcal{N}_{\mathrm{comb}}$ is a geometric term which gives the number of possible combinations in the camera for the user-defined trigger pattern.
Let's assume we have m pixels in the trigger region (it can be the whole camera). In the case of *simple multiplicity* trigger conditions for n pixels above threshold (I will call this SM_n-trigger), the geometrical term is simply:

$$\mathcal{N}_{\mathrm{comb}} = \binom{m}{n} = \frac{n!}{m!\,(m-n)!}$$

In the case of *next-neighbor* trigger conditions for n pixels above threshold (which I will call NN_n-trigger) the situation is much more complex. For a hexagonal camera or trigger region with hexagonal pixels (as is usually the case) with r rings of pixels, plus the one in the center, the number of pixels is:

$$N_{\mathrm{pix}}(r) \equiv N^r_{\mathrm{pix}} = 3r(r+1) + 1$$

with this, we have (assuming $r > 3$, in order to avoid problems) the results shown in Table 1.
For instance, let's assume we have 217 pixels (number of rings is $r = 8$). Then, for different configurations of trigger, we will have the combinatorial factors shown in Table 2.

Estimation of the Artificial Trigger Rate

Let's assume we have a trigger window of $\Delta T = 10\,\mathrm{ns} = 10^{-8}\,\mathrm{s}$. In this case, we saw that for 1 pixel

$$\langle \mathrm{LONS} \rangle = 1.10\,\mathrm{ph.e^-/\,gate}$$

We then fix a value for the single pixel threshold, q_0. Let's assume we take a value of $q_0 = 7\ \mathrm{ph.e^-/pixel/gate}$. In our trigger gate, ΔT, the mean of the Poisson distribution describing the incoming LONS photons is $\lambda = \langle \mathrm{LONS} \rangle = 1.10\,\mathrm{ph.e^-/\,gate}$. The probability of getting a charge q in one pixel due to the LONS, such that $q \geq q_0 \equiv 7\,\mathrm{ph.e^-/pixel/gate}$ is:

Trigger Condition	$\mathcal{N}_{\mathrm{comb}}$
NN_2	$3\left(2n + 2\sum_{i=n}^{2n-1} i\right) = 9n^2 + 3n$
NN_3	$2\left(\sum_{i=n+1}^{2n} i + \sum_{i=n}^{2n-1} i\right) = 6n^2$
NN_4	$3\left(2n + 2\sum_{i=n+1}^{2n-1} i\right) = 9n^2 - 3n$

Table 1: Number of combinations (combinatorial factor) for NN-triggers

Trigger Condition	$\mathcal{N}_{\rm comb}$
SM_2	23 436
SM_3	1 679 580
SM_4	89 857 530
NN_2	600
NN_3	384
NN_4	552

Table 2: Number of combinations for different trigger schemes, for the case of $r = 8$, or 217 pixels.

Table 3: Artificial Trigger Rates calculated for different trigger configurations and single pixel thresholds. In all cases a trigger region of $r = 8$ rings, or 217 pixels, has been used.

$$\mathcal{P}(q \geq q_0) = \sum_{k=q_0}^{\infty} \mathcal{P}(q = k) = \sum_{k=q_0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} = 1 - \sum_{k=0}^{q_0-1} \frac{\lambda^k}{k!} e^{-\lambda}$$

If we want to use the exact values, this last expression is the best to be used. However, if $q_0 > \lambda$ (as is the case here) we can use an expansion of the first expression, using the property of the Poisson distribution $\mathcal{P}(r+1) = [\lambda/(r+1)]\mathcal{P}(r)$

$$\begin{aligned}
\mathcal{P}(q \geq q_0) &= \sum_{k=q_0}^{\infty} \mathcal{P}(q = k) \\
&= \mathcal{P}(q = q_0) + \mathcal{P}(q = q_0 + 1) + \mathcal{P}(q = q_0 + 2) + \ldots \\
&= \mathcal{P}(q = q_0) + \frac{\lambda}{(q_0 + 1)} \mathcal{P}(q = q_0) + \frac{\lambda^2}{(q_0 + 1)(q_0 + 2)} \mathcal{P}(q = q_0) + \ldots \\
&= \left[1 + \frac{\lambda}{(q_0 + 1)} + \frac{\lambda^2}{(q_0 + 1)(q_0 + 2)} + \ldots\right] \left(\frac{\lambda^{q_0}}{q_0!} e^{-\lambda}\right)
\end{aligned}$$

We can then simply take the first two or three terms of this expansion, since the rest are going to be very small.
Let's take for our example only the first two terms. In this case:

$$\mathcal{P}(q \geq 7\,\text{ph.e}^-) \approx \left[1 + \frac{1.10}{(7+1)}\right] \left(\frac{1.10^7}{7!} e^{-1.10}\right) = 1.46 \cdot 10^{-4}$$

With tihs probability, we can calculate the rate R of a single pixel triggering

$$R = \mathcal{P}(q \geq 7\,\text{ph.e}^-)/\Delta T = 14640\,\text{Hz} \approx 15\,\text{kHz}$$

This is the *singles-rate*. Using the formula given at the beginning, we would have, for instance in the case of NN_4 trigger:

$$\text{ATR}(\text{NN}_4) = \mathcal{N}_{\rm comb}(\text{NN}_4) R^4 \Delta T^3 = 552 \cdot 14640^4 \cdot (10^{-8})^3 = 2.5 \cdot 10^{-5}\,\text{Hz}$$

In the Table 3 I show the results of these calculations for different trigger patterns and thresholds.