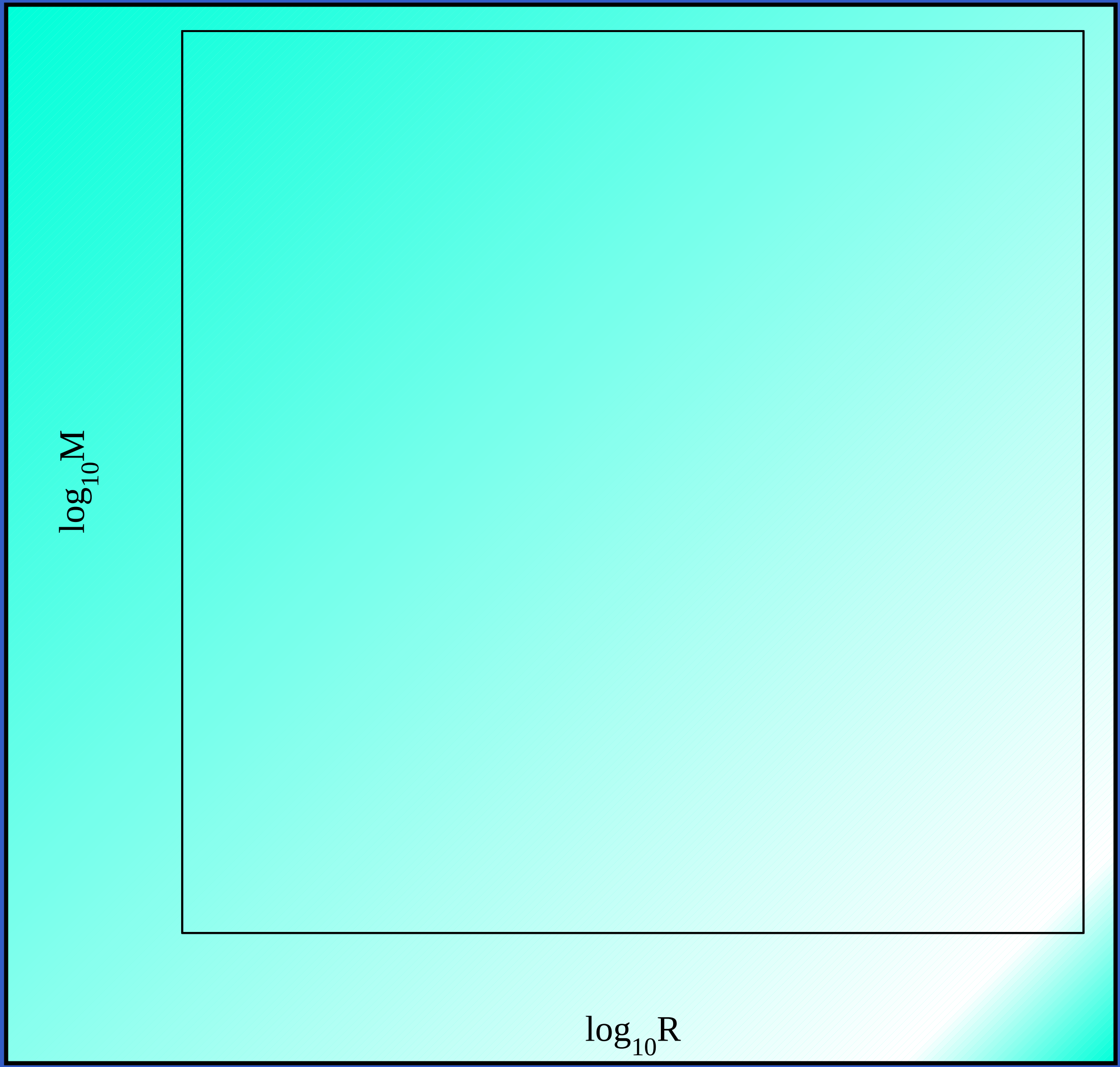
Playing with the Constants of Nature

J C González

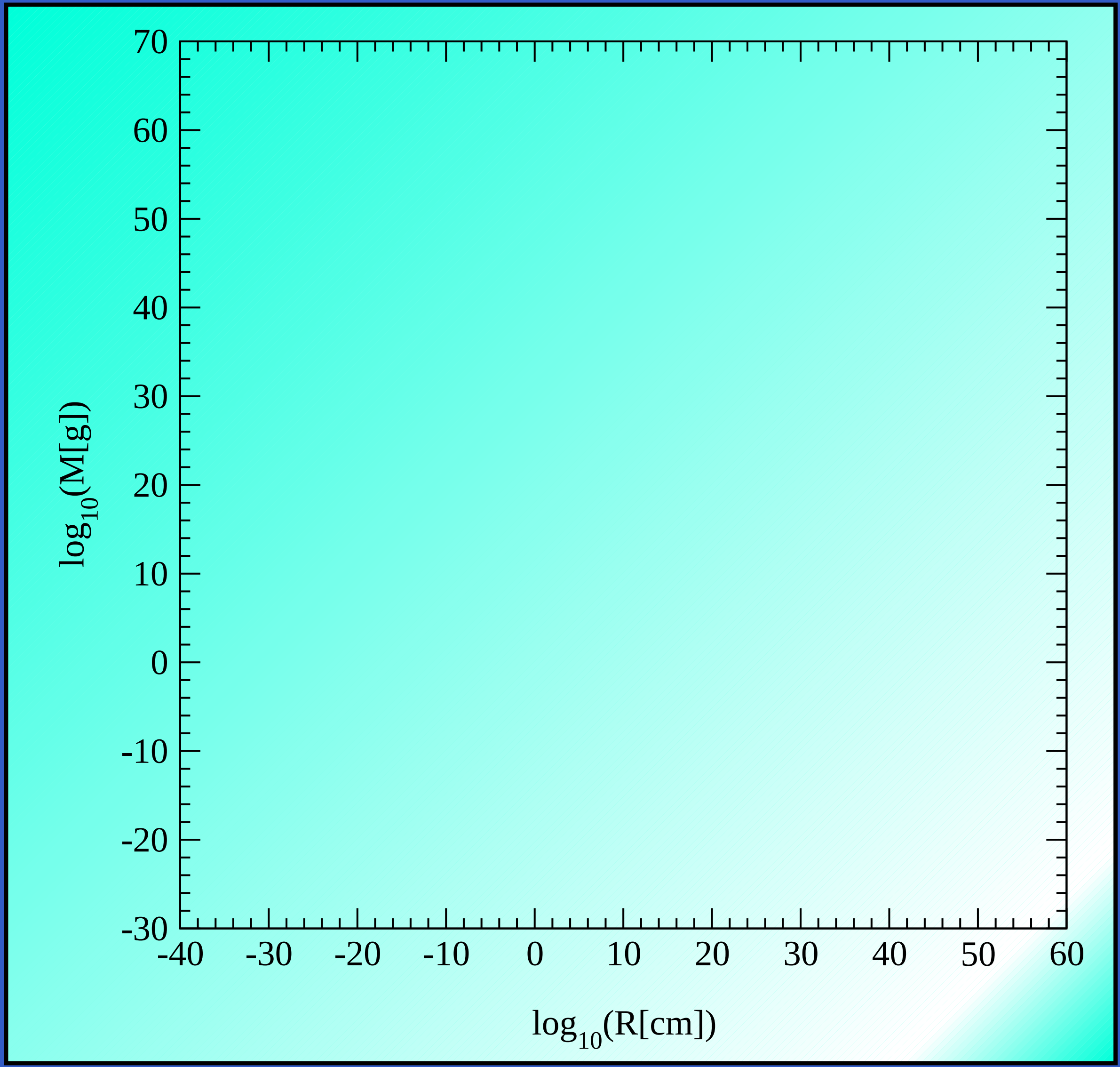
jcgonzalez@gmv.es

Global Navigation Satellite Systems Unit,
G.M.V., S.A.

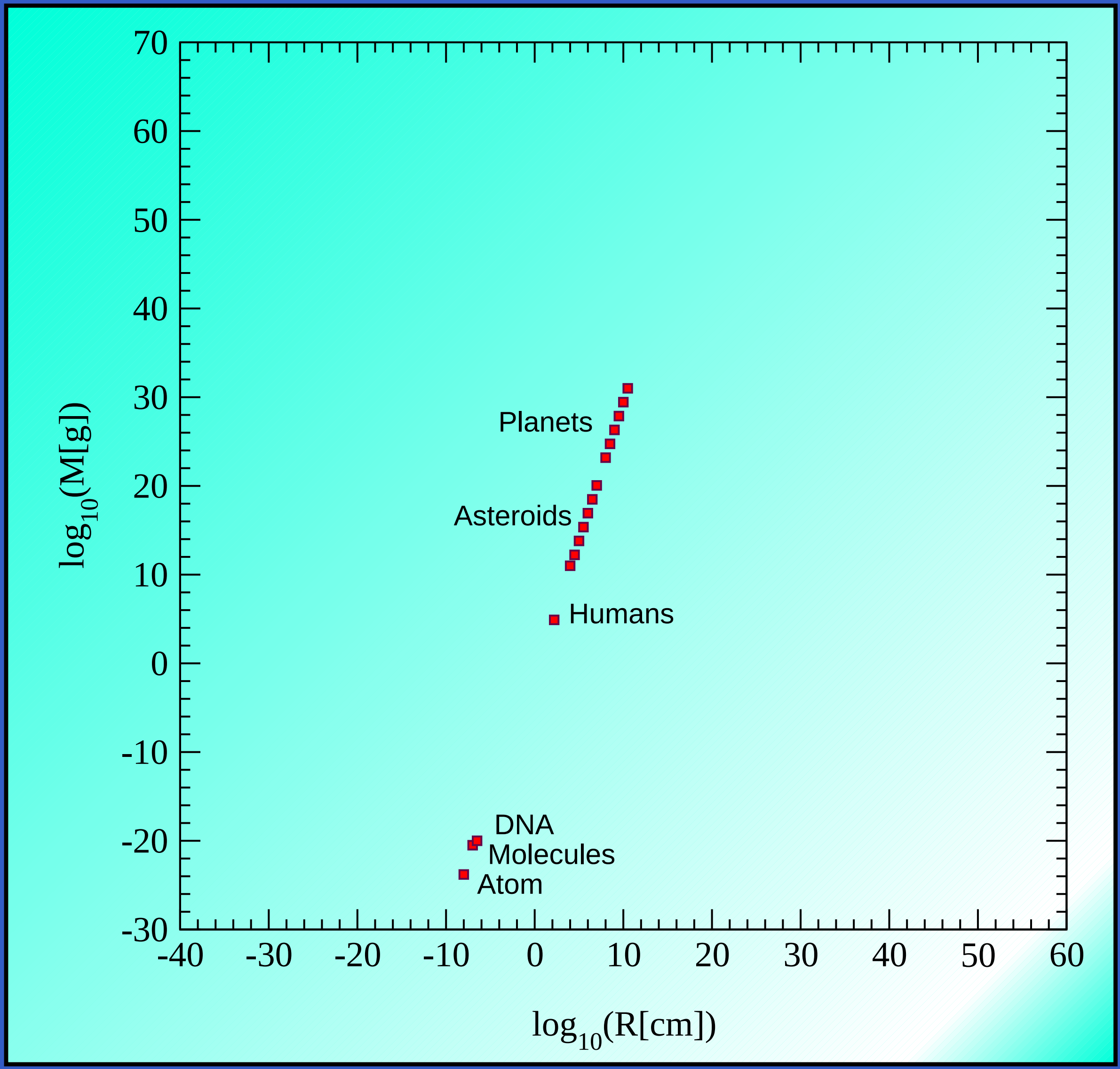
The Size of Things



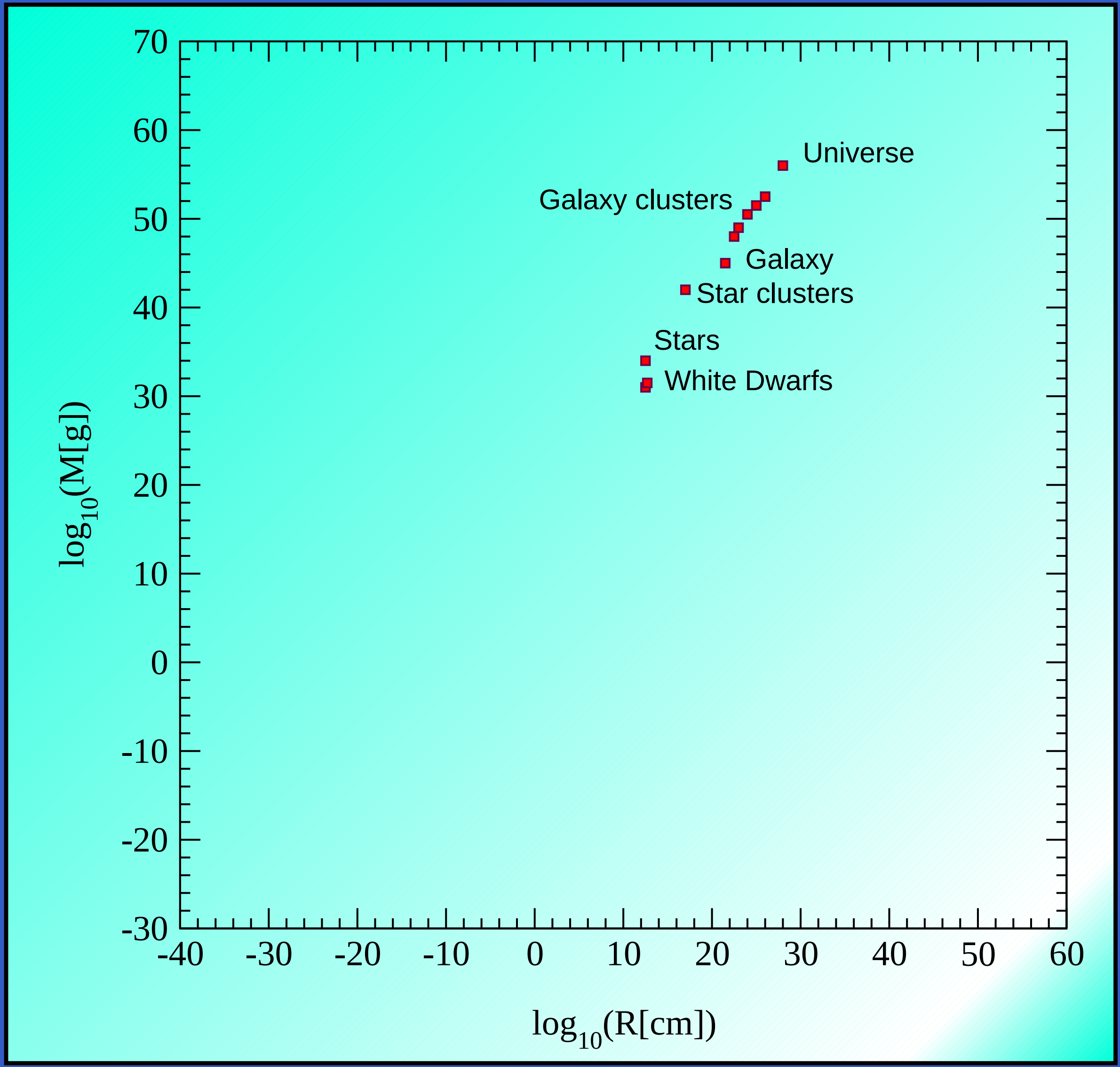
The Size of Things



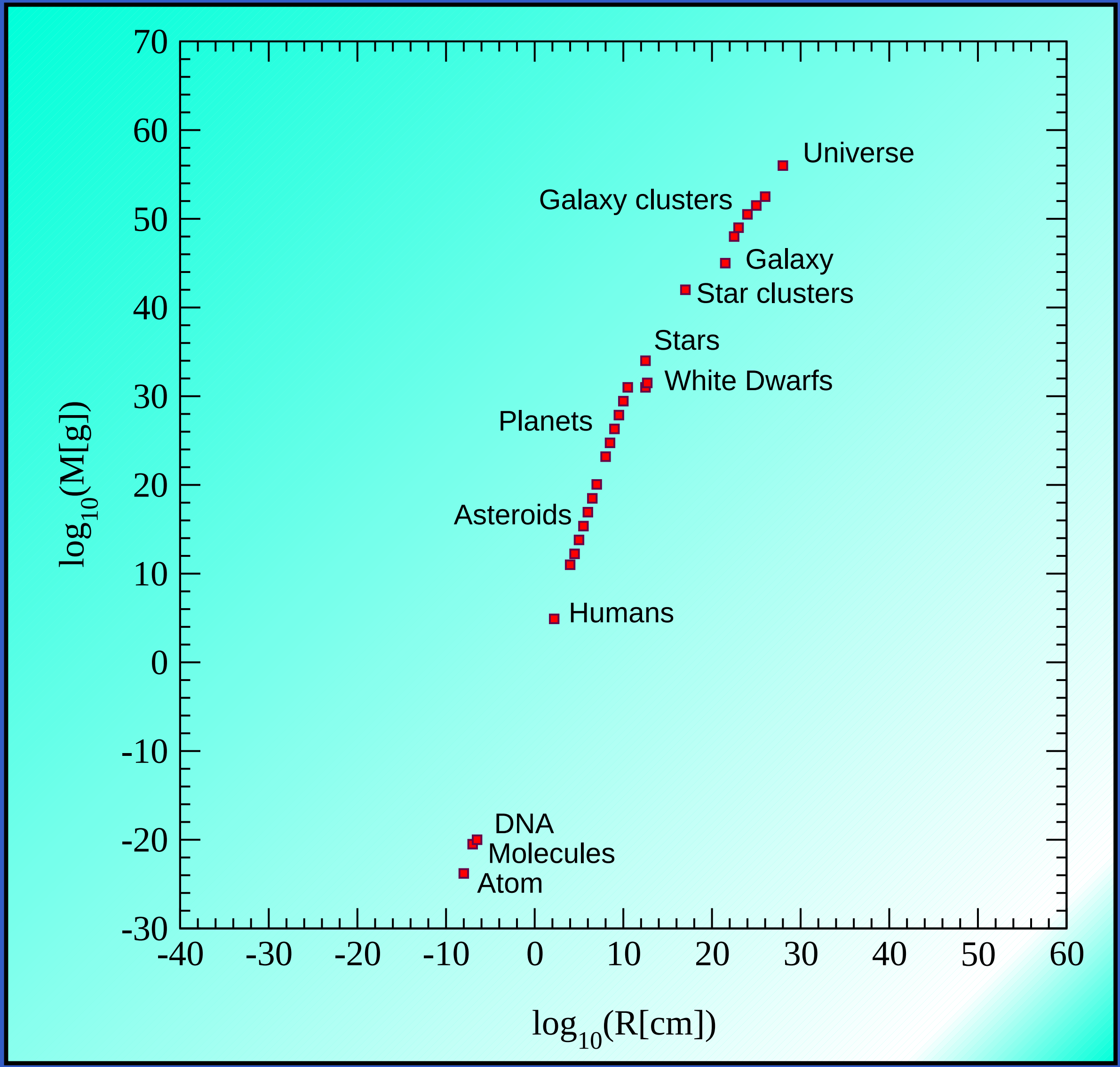
The Size of Things



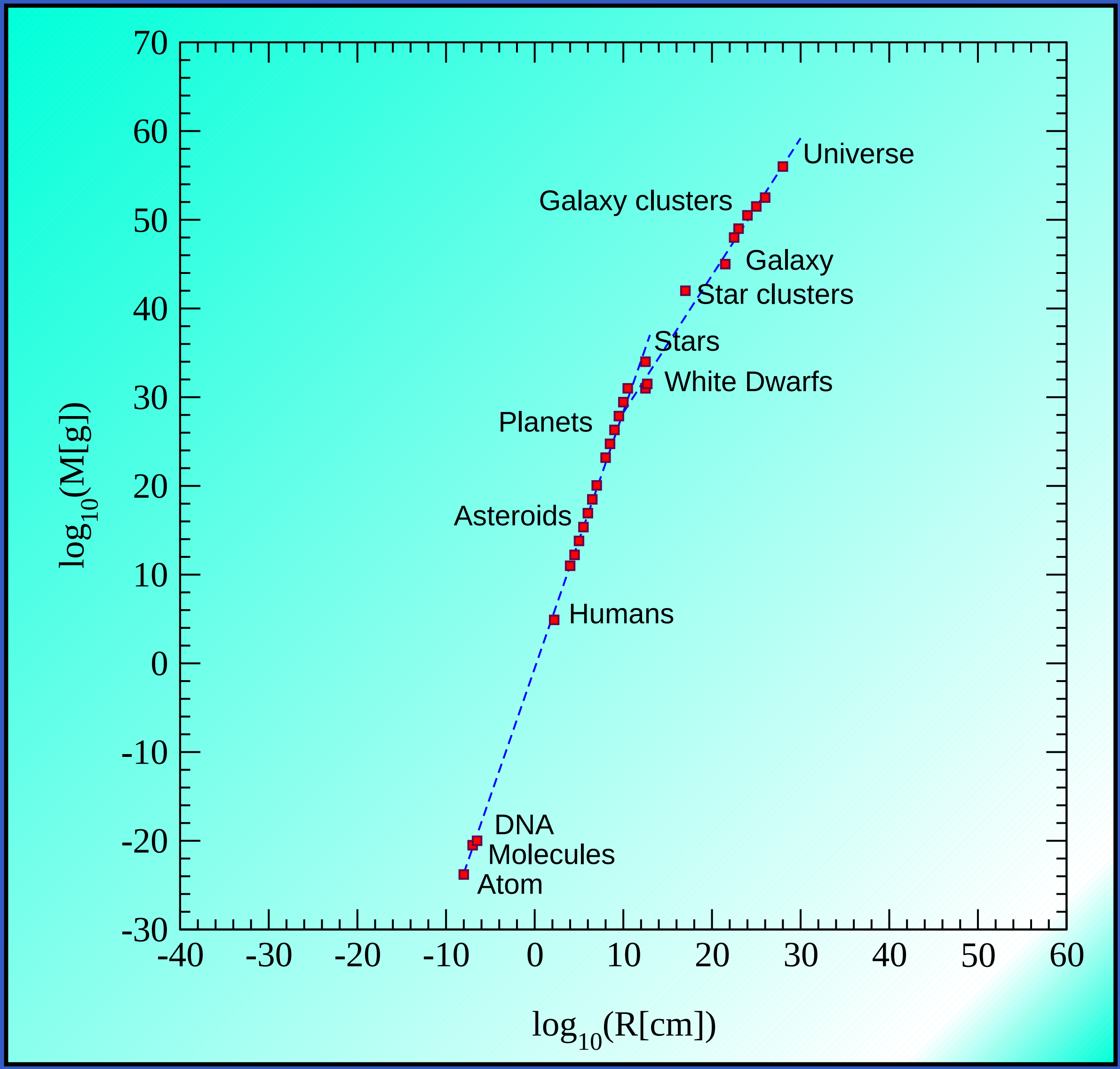
The Size of Things



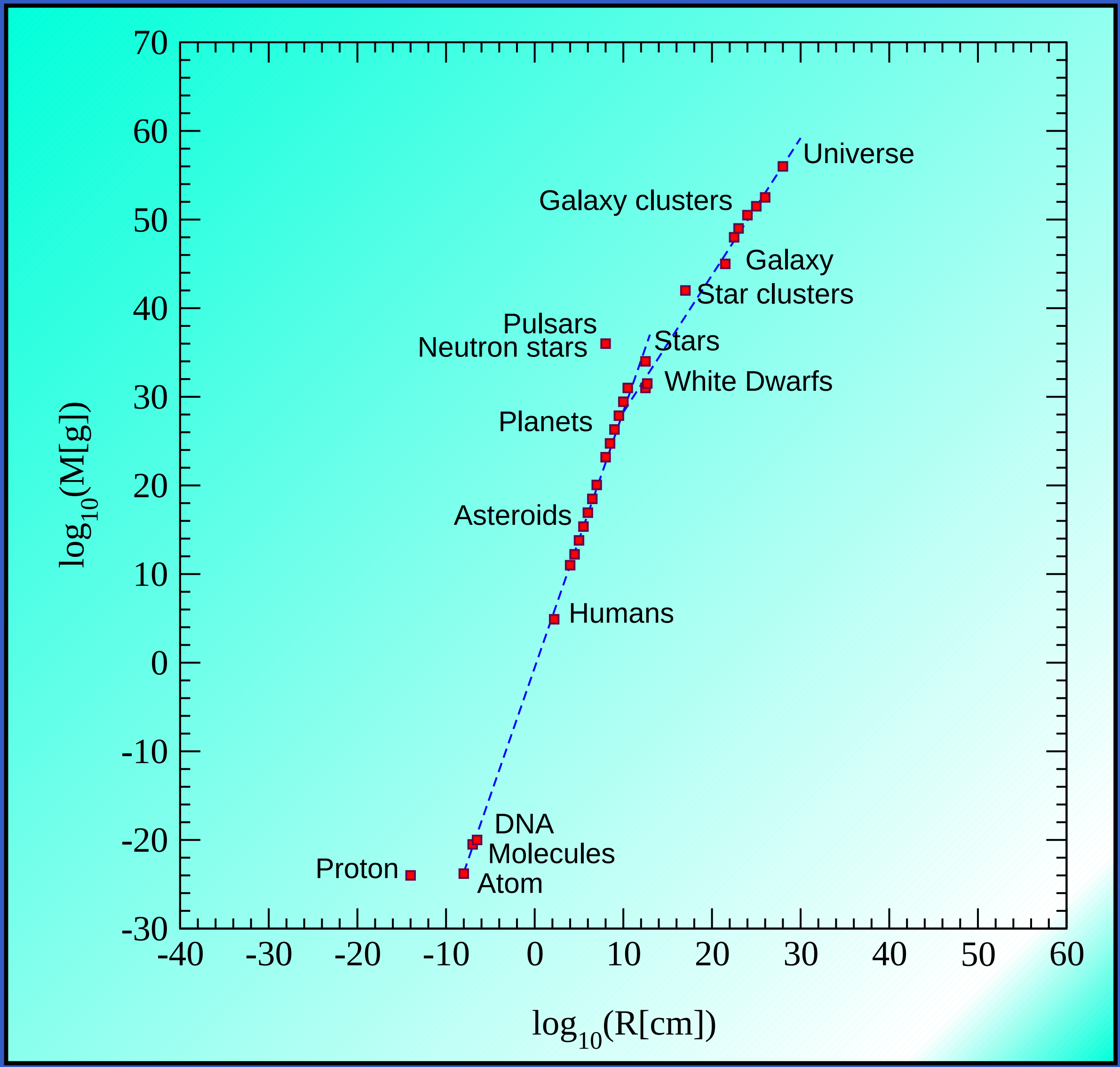
The Size of Things



The Size of Things



The Size of Things



The Size of Things (II)

Can we explain this?

The Size of Things (II)

Can we explain this?

- Sure! It's a random distribution: any visual correlation it's just a coincidence

The Size of Things (II)

Can we explain this?

- Sure! It's a random distribution: any visual correlation it's just a coincidence
- Hmm... We do see a correlation, but it's just a selection effect (invisible structures?)

The Size of Things (II)

Can we explain this?

- Sure! It's a random distribution: any visual correlation it's just a coincidence
- Hmm... We do see a correlation, but it's just a selection effect (invisible structures?)
- This illustrates the Rules of Nature

The Size of Things (II)

Can we explain this?

- Sure! It's a random distribution: any visual correlation it's just a coincidence
- Hmm... We do see a correlation, but it's just a selection effect (invisible structures?)
- This illustrates the Rules of Nature $\longrightarrow$ stability

Natural Units

G. Johnstone Stoney (1874)

$$L_{\mathrm{J}}$$

$$T_{\mathrm{J}}$$

$$M_{\mathrm{J}}$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_{\mathrm{J}}$$

$$T_{\mathrm{J}}$$

$$M_{\mathrm{J}}$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_{\mathrm{J}} = \left(\frac{Ge^2}{c^4}\right)^{1/2}$$

$$T_{\mathrm{J}}$$

$$M_{\mathrm{J}}$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_J = \left(\frac{Ge^2}{c^4}\right)^{1/2} \simeq 10^{-35}\ \mathrm{cm}$$

$$T_J$$

$$M_J$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_J = \left(\frac{Ge^2}{c^4}\right)^{1/2} \simeq 10^{-35}\,\mathrm{cm}$$

$$T_J = \left(\frac{Ge^2}{c^6}\right)^{1/2}$$

$$M_J$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_J = \left(\frac{Ge^2}{c^4}\right)^{1/2} \simeq 10^{-35}\ \text{cm}$$

$$T_J = \left(\frac{Ge^2}{c^6}\right)^{1/2} \simeq 3{\cdot}10^{-46}\ \text{s}$$

$$M_J$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_J = \left(\frac{Ge^2}{c^4}\right)^{1/2} \simeq 10^{-35}\,\text{cm}$$

$$T_J = \left(\frac{Ge^2}{c^6}\right)^{1/2} \simeq 3{\cdot}10^{-46}\,\text{s}$$

$$M_J = \left(\frac{e^2}{G}\right)^{1/2}$$

Natural Units

G. Johnstone Stoney (1874)

Units $\longrightarrow \{c, G, e\}$

$$L_{\mathrm{J}} = \left(\frac{Ge^2}{c^4}\right)^{1/2} \simeq 10^{-35}\,\mathrm{cm}$$

$$T_{\mathrm{J}} = \left(\frac{Ge^2}{c^6}\right)^{1/2} \simeq 3{\cdot}10^{-46}\,\mathrm{s}$$

$$M_{\mathrm{J}} = \left(\frac{e^2}{G}\right)^{1/2} \simeq 10^{-7}\,\mathrm{g}$$

Natural Units (II)

Max Planck (1906)

L_{P}

T_{P}

M_{P}

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\} \quad (\hbar = h/2\pi)$

L_{P}

T_{P}

M_{P}

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\} \quad (\hbar = h/2\pi)$

$$L_{\mathsf{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2}$$

$$T_{\mathsf{P}}$$

$$M_{\mathsf{P}}$$

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\} \quad (\hbar = h/2\pi)$

$$L_{\mathrm{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 10^{-33}\,\mathrm{cm} \quad (\ll 10^{-13}\,\mathrm{cm})$$

$$T_{\mathrm{P}}$$

$$M_{\mathrm{P}}$$

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\} \quad (\hbar = h/2\pi)$

$$L_{\mathrm{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 10^{-33}\,\mathrm{cm} \quad (\ll 10^{-13}\,\mathrm{cm})$$

$$T_{\mathrm{P}} = \left(\frac{G\hbar}{c^5}\right)^{1/2}$$

$$M_{\mathrm{P}}$$

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\} \quad (\hbar = h/2\pi)$

$$L_{\mathrm{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 10^{-33}\,\mathrm{cm} \quad (\ll 10^{-13}\,\mathrm{cm})$$

$$T_{\mathrm{P}} = \left(\frac{G\hbar}{c^5}\right)^{1/2} \simeq 5{\cdot}10^{-44}\,\mathrm{s} \quad (\ll 10^{-23}\,\mathrm{s})$$

$$M_{\mathrm{P}}$$

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\}$ $(\hbar = h/2\pi)$

$$L_{\mathrm{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 10^{-33}\,\mathrm{cm} \quad (\ll 10^{-13}\,\mathrm{cm})$$

$$T_{\mathrm{P}} = \left(\frac{G\hbar}{c^5}\right)^{1/2} \simeq 5{\cdot}10^{-44}\,\mathrm{s} \quad (\ll 10^{-23}\,\mathrm{s})$$

$$M_{\mathrm{P}} = \left(\frac{c\hbar}{G}\right)^{1/2}$$

Natural Units (II)

Max Planck (1906)

Units $\longrightarrow \{c, G, h\}$ $\quad (\hbar = h/2\pi)$

$$L_{\mathrm{P}} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 10^{-33}\,\mathrm{cm} \quad (\ll 10^{-13}\,\mathrm{cm})$$

$$T_{\mathrm{P}} = \left(\frac{G\hbar}{c^5}\right)^{1/2} \simeq 5{\cdot}10^{-44}\,\mathrm{s} \quad (\ll 10^{-23}\,\mathrm{s})$$

$$M_{\mathrm{P}} = \left(\frac{c\hbar}{G}\right)^{1/2} \simeq 10^{-5}\,\mathrm{g}$$

Coupling constants

Coupling constants

Electromagnetism

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Weak

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Weak

$$\alpha_{\rm W} = \frac{g_{\rm W}^2}{\hbar c} = \left(\frac{e}{\sin\theta_{\rm W}}\right)^2 \frac{1}{\hbar c} = \frac{e^2}{\sin^2\theta_{\rm W}\,\hbar c}$$

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Weak

$$\alpha_{\mathrm{W}} = \frac{g_{\mathrm{W}}^2}{\hbar c} = \left(\frac{e}{\sin\theta_{\mathrm{W}}}\right)^2 \frac{1}{\hbar c} = \frac{e^2}{\sin^2\theta_{\mathrm{W}}\,\hbar c}$$

Strong

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Weak

$$\alpha_{\mathrm{W}} = \frac{g_{\mathrm{W}}^2}{\hbar c} = \left(\frac{e}{\sin\theta_{\mathrm{W}}}\right)^2 \frac{1}{\hbar c} = \frac{e^2}{\sin^2\theta_{\mathrm{W}}\,\hbar c}$$

Strong

$$\alpha_{\mathrm{S}} = \frac{g_{\mathrm{S}}^2}{\hbar c}$$

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Weak

$$\alpha_{\mathrm{W}} = \frac{g_{\mathrm{W}}^2}{\hbar c} = \left(\frac{e}{\sin\theta_{\mathrm{W}}}\right)^2 \frac{1}{\hbar c} = \frac{e^2}{\sin^2\theta_{\mathrm{W}}\,\hbar c}$$

Strong

$$\alpha_{\mathrm{S}} = \frac{g_{\mathrm{S}}^2}{\hbar c}$$

Gravitation

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c}$$

Weak

$$\alpha_{\mathrm{W}} = \frac{g_{\mathrm{W}}^2}{\hbar c} = \left(\frac{e}{\sin\theta_{\mathrm{W}}}\right)^2 \frac{1}{\hbar c} = \frac{e^2}{\sin^2\theta_{\mathrm{W}}\,\hbar c}$$

Strong

$$\alpha_{\mathrm{S}} = \frac{g_{\mathrm{S}}^2}{\hbar c}$$

Gravitation

$$\alpha_{\mathrm{G}} = \left.\frac{Gm^2}{\hbar c}\right|_{m\equiv m_{\mathrm{N}}} = \frac{Gm_{\mathrm{N}}^2}{\hbar c}$$

Coupling constants

Electromagnetism

$$\alpha = \frac{e^2/(\hbar/mc)}{mc^2} = \frac{e^2}{\hbar c} \simeq \frac{1}{137}$$

Weak

$$\alpha_{\mathrm{W}} = \frac{g_{\mathrm{W}}^2}{\hbar c} = \left(\frac{e}{\sin\theta_{\mathrm{W}}}\right)^2 \frac{1}{\hbar c} = \frac{e^2}{\sin^2\theta_{\mathrm{W}}\,\hbar c} \simeq 0.165$$

Strong

$$\alpha_{\mathrm{S}} = \frac{g_{\mathrm{S}}^2}{\hbar c} \simeq 15$$

Gravitation

$$\alpha_{\mathrm{G}} = \left.\frac{Gm^2}{\hbar c}\right|_{m=m_{\mathrm{N}}} = \frac{Gm_{\mathrm{N}}^2}{\hbar c} \simeq 10^{-39}$$

Other constants

Fine Structure Constant & Ratio of the electron to proton mass

$$\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137} \qquad \beta = \frac{m_\mathrm{e}}{m_\mathrm{p}} = (1836.12)^{-1}$$

Other constants

Fine Structure Constant & Ratio of the electron to proton mass

$$\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137} \qquad \beta = \frac{m_\mathrm{e}}{m_\mathrm{p}} = (1836.12)^{-1}$$

Atomic Number (Z) or Atomic Weight (A)

Other constants

Fine Structure Constant & Ratio of the electron to proton mass

$$\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137} \qquad \beta = \frac{m_\mathrm{e}}{m_\mathrm{p}} = (1836.12)^{-1}$$

Atomic Number (Z) or Atomic Weight (A)

Geometrical factors ($2\pi, \ldots$)

Sample questions...

Sample questions...

How large an average planet would be?

Sample questions...

How large an average planet would be?

How large an average life form would be in such a planet?

An average planet...

An average planet...

Condition:

Stable equilibrium between the

collapsing binding energy and the

electrostatic and degeneracy pressure

An average planet...

Condition:

Stable equilibrium between the

collapsing binding energy and the

electrostatic and degeneracy pressure

Result A:

$$\text{Radius} \simeq \left(\frac{\alpha}{\alpha_{\mathrm{G}}}\right)^{1/2} \frac{a_0}{A} \simeq \frac{0.7}{A} \times 10^6\,\mathrm{km}$$

An average planet...

Condition:

Stable equilibrium between the

collapsing binding energy and the

electrostatic and degeneracy pressure

Result A:

$$\text{Radius} \simeq \left(\frac{\alpha}{\alpha_G}\right)^{1/2} \frac{a_0}{A} \simeq \frac{0.7}{A} \times 10^6\,\text{km} \simeq 10^4 - 10^5\,\text{km}$$

An average planet...

Condition:

Stable equilibrium between the

collapsing binding energy and the

electrostatic and degeneracy pressure

Result B:

$$\text{Mass} \simeq \frac{4\pi}{3} R^3 \rho_{\text{AT}} \simeq 10^{31}\,\text{g}\,\frac{1}{A^2}$$

An average planet...

Condition:

Stable equilibrium between the

collapsing binding energy and the

electrostatic and degeneracy pressure

Result B:

$$\text{Mass} \simeq \frac{4\pi}{3} R^3 \rho_{\text{AT}} \simeq 10^{31}\ \text{g} \frac{1}{A^2} \simeq 3 \times 10^{27}\ \text{g}$$

. . . with an average life form.

. . . with an average life form.

Condition:

Gravitational potential on planet surface

smaller than

Energy required for fracture

. . . with an average life form.

Condition:

Gravitational potential on planet surface

smaller than

Energy required for fracture

Result A:

$$\text{Size} \lesssim 10\left(\frac{\alpha}{10^{-3}}\right)\ \text{cm}$$

. . . with an average life form.

Condition:

Gravitational potential on planet surface

smaller than

Energy required for fracture

Result A:

$$\text{Size} \lesssim 10 \left(\frac{\alpha}{10^{-3}} \right) \text{cm} \simeq 73\,\text{cm}$$

. . . with an average life form.

Condition:

Gravitational potential on planet surface

smaller than

Energy required for fracture

Result B:

$$\text{Mass} \simeq 10^5 \text{ g}$$

Conclusion

The whole structure of the Universe can be deduced from a few fundamental constants of Nature